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Artificial gauge potentials for neutral atoms

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Workshop Hadrons and Nuclear Physics meet ultracold atoms, IHP, Paris

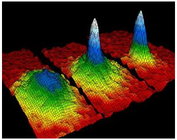
January 29, 2018

Ultracold atomic gases as many-body systems

- dilute gases but *interacting* atoms
- experimental flexibility : trapping potential, interactions, density, ...
- microscopic properties well-characterized
- well-isolated from the external world

Bose-Einstein condensates :

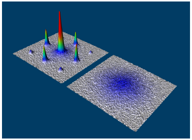
Superfluid gas
"Atom laser"



JILA, MIT, Rice (1995)

Optical lattices :

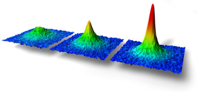
Superfluid-Mott insulator transition



Munich 2002

BEC-BCS crossover :

Condensation of fermionic pairs



JILA, MIT, ENS (2003-2004)

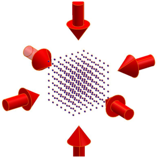
Many other examples :

- gas of impenetrable bosons in 1D,
- disordered systems, ...
- non-equilibrium many-body dynamics,

Superfluid-Mott insulator transition for bosons

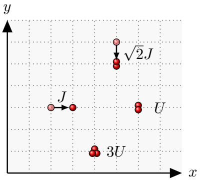
Optical lattices :

interference pattern can be used to trap atoms in a periodic structure

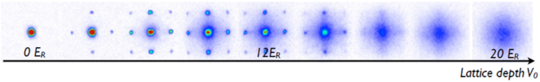


Bosons in the Bose-Hubbard regime :

1. Quantum tunneling favor delocalization
2. **Repulsive** on-site interactions favor localization



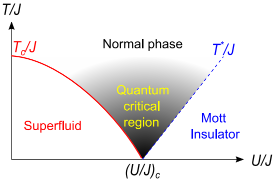
Quantum phase transition from a superfluid, Bose-condensed ground state to a Mott insulator



Greiner *et al.*, Nature 2002.

Energy/Temperature scales : nanoKelvin

Time scales $\sim 10\,\mathrm{ms}$



Vector potential $\boldsymbol{A}$ in quantum mechanics : $\hat{H} = \frac{(\hat{\boldsymbol{p}} - q\boldsymbol{A})^2}{2m}$, $\boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{B}$

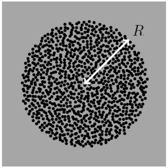
Electrons in a magnetic field exhibit many different and fascinating effects :

- Landau diamagnetism, Shubnikov-De Haas oscillations,
- Vortices in type II superconductors,
- Coherence in mesoscopic physics, ...
- Quantum Hall effect (integer and fractional)

Fractional Quantum Hall effect:

Emergence of strongly correlated phases of matter :

- incompressible liquids (gap)
- Exotic excitations with fractional charge and statistics ("anyons")
- Very similar Quantum Hall states are predicted for ultracold atomic gases [Cooper, Adv. Phys. 2008].



Laughlin state
Dubail, Read, Rezayi, PRB 2012

Key elements : flat dispersion relation and interactions

What about neutral particles (atoms) ?

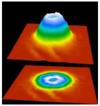
Mathematical identity between Coriolis and Lorentz force :

$$\boldsymbol{F}_{\text{Coriolis}} = m\boldsymbol{v} \times \boldsymbol{\Omega} \qquad \boldsymbol{F}_{\text{Lorentz}} = q\boldsymbol{v} \times \boldsymbol{B}$$

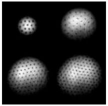
Rotation around $\boldsymbol{z}$, rotation rate Ω Magnetic field along $\boldsymbol{z}$, strength $|\boldsymbol{B}|$

Rotating superfluid atomic gases :

- Formation of quantized vortices
- Ordering into triangular vortex lattice



ENS
Madison *et al.*
PRL 2000



MIT
Abo-Shaeer *et al.*
Nature 2001

Rapidly rotating atomic gases (bosonic and fermionic) :

- Theory : strongly correlated ground states akin to fractional quantum Hall phases [Cooper, Adv. Phys. 2008]
- Experiments : so far unable to reach this regime.

Aharonov-Bohm and geometric phases

Can we explore orbital magnetism with electrically neutral atoms ?

Vector potential $\boldsymbol{A}$ in quantum mechanics :

$$\hat{H} = \frac{(\hat{\boldsymbol{p}} - q\boldsymbol{A})^2}{2m}$$

$$\boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{B}$$

$\Longleftrightarrow$

Aharonov-Bohm phase:

$$\phi_{\mathrm{AB}} = \frac{q}{\hbar} \int_{\mathcal{C}} \boldsymbol{A} \cdot d\boldsymbol{l} = \frac{q}{\hbar} \int_{\mathcal{S}} \boldsymbol{B} \cdot d\boldsymbol{\mathcal{S}}$$

What about neutral particles (atoms) ?

- Orbital magnetism can be simulated by generating geometric phases

$$\phi_{\mathrm{geo}} \equiv \frac{1}{\hbar} \int_{\mathcal{S}} (q\boldsymbol{B})_{\mathrm{eff}} \cdot d\boldsymbol{\mathcal{S}}$$

- Coherent atom-light coupling in quantum optics

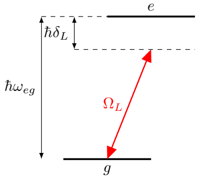
Review articles : J. Dalibard, F. Gerbier, P. Ohberg, G. Juzeliunas, RMP 2011
N. Goldman, G. Juzeliunas, P. Ohberg, I. Spielman, Rep. Progress. Physics 2014

Coherent atom-light interaction

Two-level atom and monochromatic light :

- two internal states g and e
- Hamiltonian after rotating wave approximation :

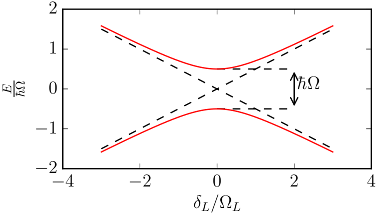
$$\hat{H}_{\mathrm{RWA}} = \begin{pmatrix} 0 & \frac{\hbar\Omega_L}{2} e^{-i\varphi} \\ \frac{\hbar\Omega_L}{2} e^{i\varphi} & -\delta_L \end{pmatrix},$$



Lowest eigenstate with energy $E_- = -\frac{1}{2}\hbar\Omega = -\frac{1}{2}\sqrt{\Omega_L^2 + \delta_L^2}$:

$$|\phi_-\rangle = \sin\left(\frac{\theta}{2}\right)|g\rangle - e^{i\varphi}\cos\left(\frac{\theta}{2}\right)|e\rangle$$

where $\cos(\theta) = \frac{\delta_L}{\Omega}$.



Adiabatic preparation :

$$\theta = \pi \longrightarrow \theta = 0$$

$$|\phi_-\rangle = |g\rangle \longrightarrow |\phi_-\rangle = -e^{i\varphi}|e\rangle$$

Harper Hamiltonian for a charged particle on a tight-binding lattice

Bulk :

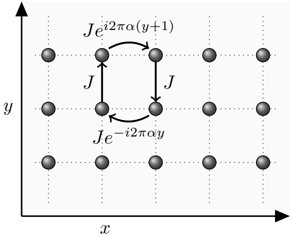
$$\hat{H} = \frac{(\hat{\boldsymbol{p}} - q\boldsymbol{A})^2}{2m}$$

$$\boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{B}$$

Tight-binding lattice :

$$H = -\sum_{\langle \boldsymbol{r}_i, \boldsymbol{r}_j \rangle} J e^{i\phi_{\mathrm{AB}}(\boldsymbol{r}_i \to \boldsymbol{r}_j)} \hat{a}_i^\dagger \hat{a}_j + \mathrm{h.c.}$$

J: single-particle tunnel energy



Landau gauge : $\boldsymbol{A} = -By\boldsymbol{e}_x$

Complex tunnel coefficients:

$$\phi_{\mathrm{AB}}(\boldsymbol{r}_i \to \boldsymbol{r}_j) = \frac{q}{\hbar}\int_{\boldsymbol{r}_i}^{\boldsymbol{r}_j} \boldsymbol{A} \cdot d\boldsymbol{l}$$

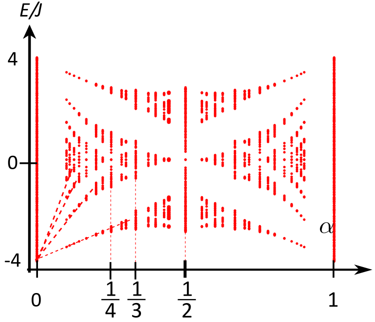
$$\alpha = \frac{|q|Bd^2}{h} = \frac{\text{Magnetic flux/unit cell}}{\text{Magnetic flux quantum}}$$

$$\alpha = \begin{cases} \sim 10^{-4} \text{ in usual solids with } \sim 50 \text{ T} \\ \sim 2\pi \text{ in solid-state superlattices or cold atoms.} \end{cases}$$

Energy spectrum vs flux :

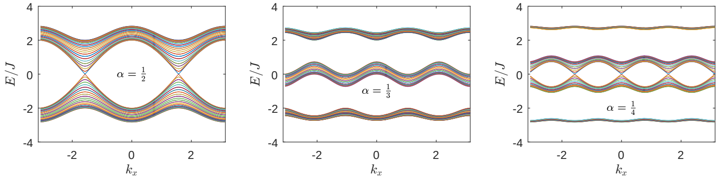
Flux per unit cell : $2\pi\alpha$

- Fragmentation of the Bloch bands
- wide gaps, flat bands



Rational flux $\alpha = p/q$:

Magnetic unit cell $(1 \times q)$: q topological bands with Chern number $\mathcal{C} \neq 0$:



Floquet approach

- Fast modulation in the Hamiltonian at Ω, induced "micromotion" at the same frequency
- Slow ("secular") motion governed by an effective Hamiltonian $\overline{H}_{\text{eff}}$

$$\hat{H}_{\text{eff}} \approx \frac{\Omega}{2\pi} \int_0^{2\pi/\Omega} \hat{H}(t) dt$$

  - Shaken optical lattices Pisa, Hamburg, Zürich, Chicago, ...
  - Sliding optical lattices : Munich, MIT

"Quantum optics" approach

- internal states coupled by one or two-photon transitions
- affects the *external* state as well due to the recoil effect
  - Adiabatic dressing (bulk systems)NIST 2008
  - Synthetic dimensions NIST, Florence 2015
  - laser-induced tunneling in optical lattices Rustekovski, Dunne, Javanainen (1998-2000); Jaksch and Zoller (2003); Gerbier and Dalibard (2010).

- Most of the schemes (proposed and realized) rely on optical lattices,
- All involve breaking some symmetry of the "bare" lattice Hamiltonian and projection onto a low-energy subspace.

Topological phases of matter

Phase of matter characterized by one (or more) integer-valued *Topological invariants* linked to certain physical properties.

- gapped phase (band gap in fermionic insulators)
- robustness with respect to microscopic changes (as long as the gap do not close)
- Bulk-edge correspondence :
  - Edge-state channels in a bounded geometry, which carry current (or heat or ...)
  - Protected against scattering
  - Number of channels determined by topological properties of the bulk

Materials/Phases that are insulating in the bulk, but with a perfect (or very good...) conducting surface

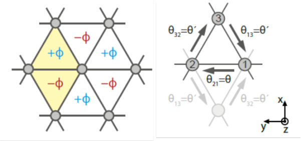
Note that these phases usually escape the standard classification of phases in condensed matter using the concept of order parameter and Landau theory.

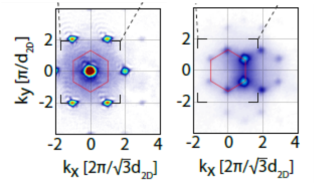
Snapshots of experiments using the Floquet approach

Lattice shaking :

$$V_{\mathrm{lat}}(x,t) = -V_0 \cos\left[k_L(x - x_0(t))\right]$$

Staggered flux in hexagonal lattices :



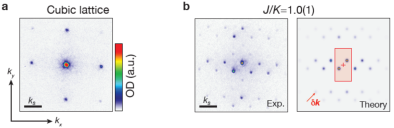


Figures from J. Struck *et al.*, Hamburg
Also Pisa, Chicago, Zürich, Munich, ...

Sliding lattice :

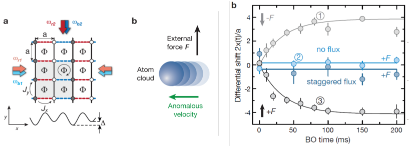
$$W(x,t) = W_0 \cos\left[\delta\boldsymbol{k}\cdot\boldsymbol{r} - \Delta\omega t\right]$$

Staggered flux in square lattices :



M. Aidelsburger *et al.* (Munich), 2012

Uniform flux in square lattices :



M. Aidelsburger *et al.* (Munich), 2015
Similar experiments at MIT (Ketterle group)

Quantized Hall Conductivity and Chern Number

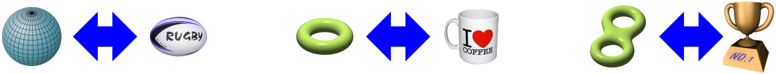
Hall conductivity : current along x flowing in response to an applied electric field E_y along y, $j_x = \sigma_H E_y$

Linear response (Kubo formalism) for fermionic insulators (Fermi energy inside a gap) :

$$\sigma_H = \frac{e^2}{h} \times \sum_{\epsilon_n(\boldsymbol{k})<E_F} \mathcal{C}_n$$

$\mathcal{C}_n$: Chern number of the nth band
Necessarily an integer !
Topological invariant that do not change by smooth deformation of the Hamiltonian

Topological classification of surfaces in 3D space :
Surfaces in ordinary 3D space can be classified by their *genus* ($\equiv$ number of handles) :



Mapping $(x, y) \rightarrow \mathcal{S}(x, y)$

Gauss-Bonnet formula : $\int_{\mathcal{S}} \mathcal{G} = 4\pi(1-g)$

$\mathcal{G}$: Gaussian curvature

Mapping $(k_x, k_y) \rightarrow \hat{H}(\boldsymbol{k})$

Chern formula : $\int_{\mathrm{BZ}} \mathcal{B}(\boldsymbol{k}) = \mathcal{C}$

$\mathcal{B}$: Berry curvature

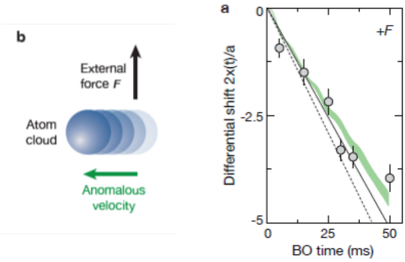
Measuring the Chern number of Hosftadter bands with hot bosons

- apply additional “electric field” $V = \boldsymbol{F} \cdot \boldsymbol{r}$ [$\boldsymbol{F}$: constant force]
- $\boldsymbol{\mathcal{B}} \neq 0$: additional deflection of the c.o.m. transverse to $\boldsymbol{F}$, $\overline{x} \propto F\mathcal{C}t$

Transverse displacement after one period $T_B = \mathcal{C} \times$ lattice spacing

Munich experiment : Aidelsburger *et al.*, Nat. Phys. 2014

- Hofstadter lattice with $\alpha = 1/4$
- bandwidth $\ll k_B T \ll$ band gap



$$P_0 \approx 1 \implies \mathcal{C}_0 \approx 0.9(1)$$

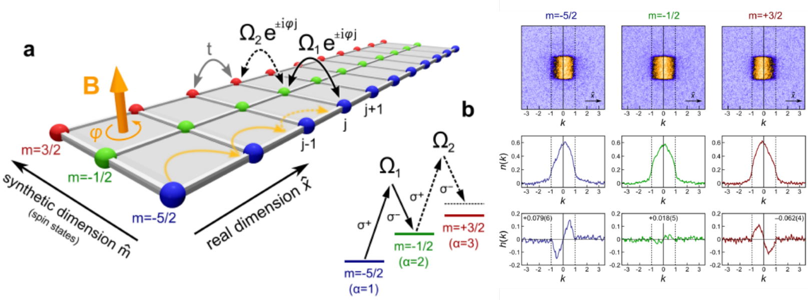
Direct measurements of Berry curvature: Fläschner *et al.*, Science 2016

see also : Duca *et al.*, Science 2014.

Synthetic dimensions

Internal degree of freedom (Zeeman states) $\equiv$ sites of a fictitious lattice

Two-photon Raman transition $\equiv$ hopping in a tight-binding model



References : M. Mancini *et al.*; B. Stuhl *et al.*, Science 2015
Figure taken from the LENS experiment (Mancini *et al.*)

- Allows to study four-dimensional physics !
- System size necessarily small in the synthetic dimension
- Interactions are local in real space, of “infinite range” in the synthetic dimension

Bosons in topological bands

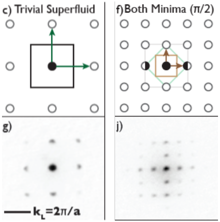
Generally we expect bosons at low T to condense into the single-particle minima.

$\alpha = 0, 1/2$: BEC observed

2D: Struck *et al.*, Nature Physics 2012

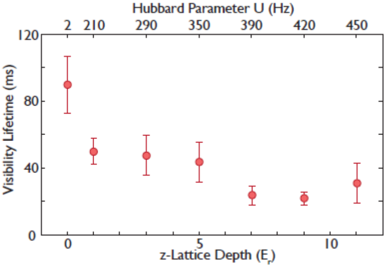
3D :Kennedy *et al.*, Nature Physics 2015

Experiments with $\alpha \neq 0, 1/2$: BEC does not survive, lowest band (almost) uniformly filled ($T \gg$ bandwidth)



Kennedy *et al.* (MIT)

- Heating generally observed in shaking experiments (timescale $\sim 50\,\text{ms}$)
- redistribution of the "micromotion" energy by collisions
- possibly off-resonant transfer from the ground to higher bands ?
- Currently under active investigation in several experimental groups



Summary

- Realization of topological band structures with cold atoms
- Two broad classes of approaches :
  - Floquet methods with rapidly modulated potentials,
  - "Quantum optics" methods using coherent manipulation of the atom internal degrees of freedom.
- Experiments have demonstrated *single-particle effects* tied to the topological band structure.
- The goal of studying strongly interacting topological phases is still elusive : heating issues encountered in experiments must be resolved.
- Non-Abelian gauge potentials can also be realized using similar techniques :
  - Spin-orbit coupling, 2D or 3D Kane and Hasan, RMP 2010
  - Topological superfluids : Pairing interaction required
    - $p-$wave order parameter
    - zero-energy modes behaving as Majorana fermions

Towards topological insulators with cold atoms ?

- fermionic band insulator with the Fermi energy inside a gap
- topological band structure

Chern insulators : topological invariant= Chern number

- integer Quantum Hall states (yet to be demonstrated with cold atoms)
- Haldane insulator (realized by the ETH Zürich group)

Many more possibilities with non-Abelian gauge potentials :

- Spin-orbit coupling, 2D or 3D Kane and Hasan, RMP 2010
- Topological superfluids : Pairing interaction required
  - $p-$wave order parameter
  - zero-energy modes behaving as Majorana fermions

Relevant parameter :

$$\nu = \frac{\text{atomic density}}{\text{flux per unit cell}} = \frac{n}{\alpha}$$

- Analogue of continuum ($\equiv$ Lowest Landau level) states exist.

  Example : Laughlin states
  - fermions : $\nu = \frac{1}{3}, \cdots$
  - bosons : $\nu = \frac{1}{2}, \cdots$

  Sorensen *et al.*, PRL 2005
  Hafezi *et al.*, PRA 2007, EPL 2008
  Palmer, Klein, Jaksch, PRL 2006; PRA 2008
  Möller, Cooper, PRL 2009 **...**

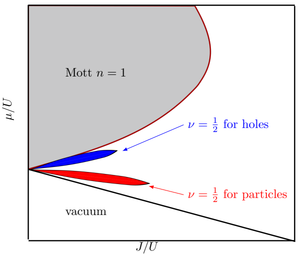
- Many possible states without continuum counterparts [Möller and Cooper, PRL 2009].

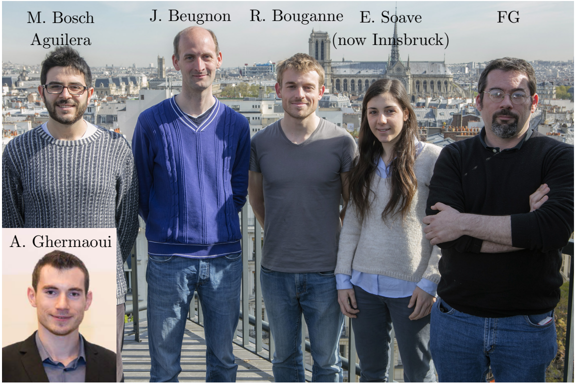
Example for $\alpha = \frac{1}{5}$:
- Laughlin state for particles at $n = \frac{1}{10}$
- Laughlin state for holes at $n = 1 - \frac{1}{10}$

Gaps are small :

at most $\sim 0.1J$ for the $\nu = \frac{1}{2}$ bosonic Laughlin state
[Hafezi et al., PRA 2007]

Narrow slices in the global phase diagram





Former members :

Q. Beaufils

A. Dareau

D. Doering

M. Scholl

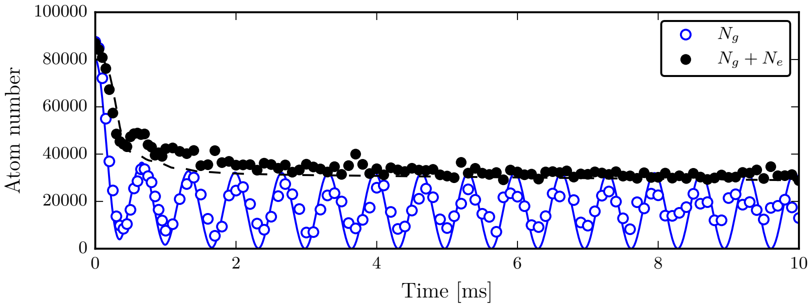
E. Soave

Some recent works :

- *Revealing the Topology of Quasicrystals with a Diffraction Experiment*
  [Dareau *et al.*, Phys. Rev. Lett. **119**, 21530 (2017).

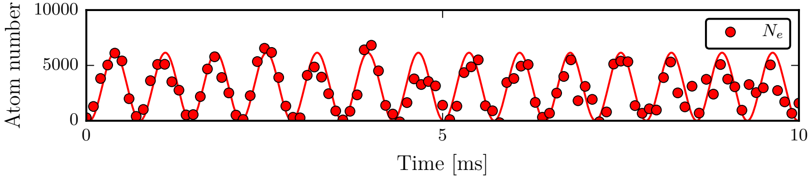
- *Clock spectroscopy of interacting bosons in deep optical lattices*
  [Bouganne *et al.*, New J. Phys. **19**, 113006 (2017).

Strong driving: Rabi oscillations of a BEC in the optical domain



Total atom number : $N \approx 8 \times 10^4$

For much lower atom numbers : $N \approx 8 \times 10^3$

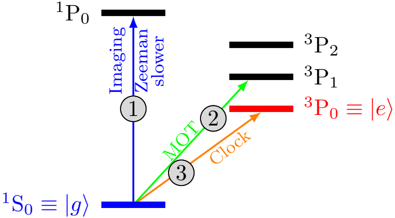


Optical atomic clock technology to study many-body phenomena

Level structure of Ytterbium

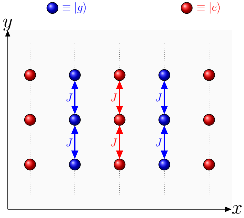
- "clock" transition 3 : $J = 0 \to J' = 0$
- virtually no spontaneous emission

  $\to$ coherent manipulation



State-dependent 2D optical lattice

- y lattice at "magic" wavelength : $V_e(y) = V_g(y)$
- x lattice at "anti-magic" wavelength : $V_e(x) = -V_g(x)$



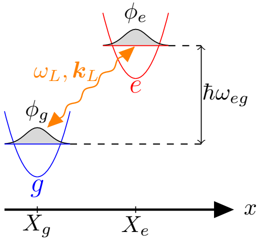
- regular tunneling along y
- supressed tunneling along x

$\lambda_x = 610$ nm
$\lambda_y = 759.5$ nm

Laser-induced tunneling in a state-dependent optical lattice

Proposal for alkali atoms in [Jaksch and Zoller, NJP 2003]

- two internal states g and e
- state-dependent potential confining the atoms at **distinct places** depending on their internal state



Coupling laser $|g; \boldsymbol{R}_g\rangle \to |e; \boldsymbol{R}_e\rangle$:

$$\langle e; \boldsymbol{R}_e | \hat{V}_{AL} | g; \boldsymbol{R}_g \rangle \propto e^{i\boldsymbol{k}_L \cdot \frac{\boldsymbol{R}_g + \boldsymbol{R}_e}{2}}$$

Not enough to get $\oint \boldsymbol{A} \cdot \boldsymbol{dl} \neq 0$, but good starting point !

Coherent driving of the clock transition : Bouganne *et al.*, NJP 2017